SD Codes: Erasure Codes Designed for How Storage Systems Really Fail

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James S. Plank
Department of Electrical Engineering and Computer Science
University of Tennessee
plank@cs.utk.edu

Mario Blaum, James L. Hafner
IBM Research Division, Almaden Research Center
mblaum@hotmail.com, hafner@us.ibm.com

Abstract

Traditionally, when storage systems employ erasure codes, they are designed to tolerate the failures of entire disks. However, the most common types of failures are latent sector failures, which only affect individual disk sectors, and block failures which arise through wear on SSD’s. This paper introduces SD codes, which are designed to tolerate combinations of disk and sector failures. As such, they consume far less storage resources than traditional erasure codes. We specify the codes with enough detail for the storage practitioner to employ them, discuss their practical properties, and detail an open-source implementation.

1 Introduction

Storage systems have grown to the point where failures are commonplace and must be tolerated to prevent data loss. All current storage systems that are composed of multiple components employ erasure codes to handle disk failures. Examples include commercial storage systems from Microsoft [10, 22], IBM [24], Netapp [12], HP [29], Cleversafe [42] and Panasas [52], plus numerous academic and non-commercial research projects [8, 11, 20, 25, 26, 43, 47, 48, 51, 54]. In all of these systems, the unit of failure is the disk. For example, a RAID-6 system dedicates two parity disks to tolerate the simultaneous failures of any two disks in the system [4, 12]. Larger systems dedicate more disks for coding to tolerate larger numbers of failures [10, 42, 51].

Recent research, however, studying the nature of failures in storage systems, has demonstrated that failures of entire disks are relatively rare. The much more common failure type is the Latent Sector Error or Undetected Disk Error where a sector on disk becomes corrupted, and this corruption is not detected until the sector in question is subsequently accessed for reading [2, 14, 19].

Additionally, storage systems are increasingly employing solid state devices as core components [3, 18, 23, 33]. These devices exhibit wear over time, which manifests in the form of blocks becoming unusable as a function of their number of overwrites, and in blocks being unable to hold their values for long durations.

To combat block failures, systems employ scrubbing, where sectors and blocks are proactively probed so that errors may be detected and recovered in a timely manner [1, 14, 35, 44, 45]. The recovery operation proceeds from erasure codes — a bad sector on a device holding data is reconstructed using the other data and coding devices. A bad sector on a coding device is re-encoded from the data devices.

Regardless of whether a system employs scrubbing, a potentially catastrophic situation occurs when a disk fails and a block failure is discovered on a non-failed device during reconstruction. It is exactly this situation which motivated companies to switch from RAID-5 to RAID-6 systems in the past decade [12, 14].

However, the RAID-6 solution to the problem is overkill: two entire disk’s worth of storage are dedicated to tolerate the failure of one disk and one sector. In effect, an entire disk is dedicated to tolerate the failure of one sector.

In this paper, we present an alternative erasure coding methodology. Instead of dedicating entire disks for fault-tolerance, we dedicate entire disks and individual sectors. For example, in the RAID-6 scenario above, instead of dedicating two disks for fault-tolerance, we dedicate one disk and one sector per stripe. The system is then fault-tolerant to the failure of any single disk and any single sector within a stripe.

We name the codes “SD” for “Sector-Disk” erasure codes. They have a general design, where a system composed of \( n \) disks dedicates \( m \) disks and \( s \) sectors per stripe to coding. The remaining sectors are dedicated to data. The codes are designed so that the simultaneous failures of any \( m \) disks and any \( s \) sectors per stripe may be tolerated without data loss.

In this paper, we present these codes for the storage
practitioner and researcher. We do not dwell on the codes’ theoretical aspects, but instead present them in such a way that they may be used in practical storage settings. To that end, we have written an SD encoder and decoder in C, which we post as open source code.

2 System Model and Nomenclature

We concentrate on a stripe of a storage system, as pictured in Figure 1. The stripe is composed of \( n \) disks, each of which holds \( r \) sectors. We may view the stripe as a \( r \times n \) array of sectors; hence we call \( r \) the number of rows. Of the \( n \) disks, \( m \) are devoted exclusively to coding. In the remaining \( n - m \) disks, \( s \) additional sectors are also devoted to coding. These sectors are distributed evenly among the \( n - m \) disks. By convention, we picture them in the bottom rows of the array.

![Figure 1: A stripe in a storage system with \( n \) total disks. Each disk is partitioned into \( r \) rows of sectors. Of the \( n \) disks, \( m \) are devoted exclusively to coding, and \( s \) additional sectors are devoted to coding.](image)

While we refer to the basic blocks of the array as sectors, they may comprise multiple sectors. For the purposes of coding, we will also consider them to be \( w \)-bit symbols, where \( w \) is a parameter of the erasure code, typically 8, 16 or 32, so that sectors may be partitioned evenly into symbols. As such, we will use the terms “block,” “sector” and “symbol” interchangeably.

Storage systems may be partitioned into many stripes, where the identities of the coding disks change from stripe to stripe to alleviate bottlenecks. The identities may be rotated as in RAID-5, or performed on an ad-hoc, per-file basis as in Panasas [52]. Blocks may be small, as in the block store of Cleverson’s distributed file system [42] or large as in Google FS [15]. A thorough discussion of the performance implications of mapping erasure code symbols to disk blocks may be found in recent work by Khan et al [25].

3 Arithmetic for Coding

To perform encoding and decoding, each coding symbol is defined to be a linear combination of the data symbols. The arithmetic employed is Galois Field arithmetic over \( w \)-bit symbols, termed \( GF(2^w) \). This is the standard arithmetic of Reed-Solomon coding. Addition in \( GF(2^w) \) is equivalent to the bitwise exclusive-or operation. Multiplication is more complex, but for the purposes of this paper, we do not need to consider how it is implemented. Please see the tutorial by Plank [39] and further material by Greenan et al [16] and Luo et al [31] for details on implementing Galois Field arithmetic in software. Open source implementations of Galois Field arithmetic are abundant (e.g. [34, 37, 41]). Although Plank et al demonstrated that erasure codes based on Galois Fields are roughly four times slower than those based on XOR operations [40], new processor architectures that implement Intel’s SSE2 extensions allow us to implement \( GF(2^8) \) and \( GF(2^{16}) \) at rates that are cache, rather than CPU-limited. \( GF(2^{32}) \) is only marginally slower.

Typical presentations of erasure codes based on Galois Field arithmetic use terms like “irreducible polynomials” and “primitive elements” to define the codes [5, 6, 16]. In our work, we simply use numbers between 0 and \( (2^w - 1) \) to represent \( w \)-bit symbols, and assume that the codes are implemented with a standard Galois Field arithmetic library such as those listed above. Our goal is to be practical rather than overly formal.

4 SD Code Specification

SD codes are defined by six parameters listed below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>The total number of disks</td>
</tr>
<tr>
<td>( m )</td>
<td>The number of coding disks</td>
</tr>
<tr>
<td>( s )</td>
<td>The number of coding sectors</td>
</tr>
<tr>
<td>( r )</td>
<td>The number of rows per stripe</td>
</tr>
<tr>
<td>( GF(2^w) )</td>
<td>The Galois field</td>
</tr>
<tr>
<td>( A = {a_0, \ldots, a_{m+s-1}} )</td>
<td>Coding coefficients</td>
</tr>
</tbody>
</table>

We label the disks \( D_0 \) through \( D_{n-1} \) and assume that disks \( D_{n-m} \) through \( D_{n-1} \) are the coding disks. There are \( nr \) blocks in a stripe, and we label them in two ways. The first assigns subscripts for the row and the disk, so that disk \( D_i \) holds blocks \( b_{0,i} \) through \( b_{r-1,i} \). This labeling is the one used in Figure 1. The second simply numbers the blocks consecutively, \( b_0 \) through \( b_{nr-1} \).
The mapping between the two is that block \( b_{j,i} \) is also block \( b_{j,n+i} \). Figure 2 shows the same stripe as Figure 1, except the blocks are labeled with their single subscripts.

\[
D_0 \quad D_1 \quad D_2 \quad D_3 \quad D_4
\]  
\[
\begin{array}{cccccc}
 b_2 & b_3 & b_4 & b_5 & b_6 & b_7 \\
 b_8 & b_9 & b_{10} & b_{11} & b_{12} & b_{13} \\
 b_{14} & b_{15} & b_{16} & b_{17} & b_{18} & b_{19} \\
 b_{20} & b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\
 \end{array}
\]

Figure 2: The same stripe as Figure 1, except blocks are labeled with single subscripts.

Instead of using a generator matrix as in Reed Solomon codes, we employ a set of \( mr + s \) equations, each of which sums to zero. The first \( mr \) of these are labeled \( C_{j,x} \), with \( 0 \leq x < m \) and \( 0 \leq j < r \). They are each the sum of exactly \( n \) blocks in a single row of the array:

\[
C_{j,x} : \sum_{i=0}^{n-1} a_x^{i+j} b_{j,i} = 0.
\]

The remaining \( s \) equations are labeled \( S_x \) with \( 0 \leq x < s \). They are the sum of all \( nr \) blocks:

\[
S_x : \sum_{i=0}^{rn-1} a_x^{i} b_{i} = 0.
\]

Intuitively, one can consider each block \( b_{j,i} \) on a coding disk to be governed by \( C_{j,x} \), and each additional coding block governed by a different \( S_x \). However, the codes are not as straightforward as, for example, classic Reed-Solomon codes, where each coding block is the linear combination of the data blocks. Instead, unless \( m \) equals one, every equation contains multiple coding blocks, which means that encoding must be viewed as a special case of decoding.

A concrete example helps to illustrate and convey some intuition. Figure 3 shows the ten equations that result when the stripe of Figures 1 and 2 is encoded with \( A = \{1, 2, 4, 8\} \). The figure is partitioned into four smaller figures, which show the encoding with each \( a_i \). The top two figures show the equations \( C_{j,0} \) and \( C_{j,1} \), which employ \( a_0 \) and \( a_1 \) respectively. Each equation is the sum of six blocks. The bottom two figures show \( S_0 \) and \( S_1 \), which are each the sum of all 24 blocks.

As mentioned above, encoding with these equations is not a straightforward activity, since each of the ten equations contains at least two coding blocks. Thus, encoding is viewed as a special case of decoding — when the two coding disks and two coding sectors fail.

The decoding process is straightforward linear algebra. When a collection of disks and sectors fail, their values of \( b_i \) are considered unknowns. The non-failed blocks are known values. Therefore, the equations become a linear system with unknown values, which may be solved using Gaussian Elimination.

For example, suppose we want to encode the system of Figure 3. To do that, we assume that disks 4 and 5 have failed, along with blocks \( b_{20} \) and \( b_{21} \). The ten equations are rearranged so that the failed blocks are on the left and the nonfailed blocks are on the right. Since addition is equivalent to exclusive-or, we may simply add a term to both sides of the equation to move it from one side to another. For example, the four equations for \( C_{j,1} \) become:

\[
\begin{align*}
2^4 b_4 + 2^5 b_5 &= b_0 + 2b_1 + 2^2 b_2 + 2^3 b_3 \\
2^10 b_{10} + 2^{11} b_{11} &= 2^6 b_0 + 2^7 b_1 + 2^8 b_8 + 2^9 b_9 \\
2^{16} b_{16} + 2^{17} b_{17} &= 2^{12} b_{12} + 2^{13} b_{13} + 2^{14} b_{14} + 2^{15} b_{15} \\
2^{20} b_{20} + 2^{21} b_{21} + 2^{22} b_{22} + 2^{23} b_{23} &= 2^{18} b_{18} + 2^{19} b_{19} \\
\end{align*}
\]

We are left with ten equations and ten unknowns, which we then solve with Gaussian Elimination or matrix inversion.

This method of decoding is a standard employment of a Parity Check Matrix \([32, 38]\). This matrix (conventionally labeled \( H \)) contains a row for every equation and a
column for every block in the system. The element in row \(i\) column \(j\) is the coefficient of \(b_j\) in equation \(i\). The vector \(B = \{b_0, b_1, \ldots, b_{n_r-1}\}\) is called the codeword, and the \(mr + s\) equations are expressed quite succinctly by the equation \(HB = 0\).

5 The SD Condition and Constructions

A code specified by the parameters above is SD if it decodes all combinations of \(m\) disks and \(s\) sectors (blocks). In other words, when the \(mr + s\) decoding equations are created, the Gaussian Elimination proceeds successfully. Put another way, the sub-matrix of the Parity Check Matrix composed of the columns that correspond to failed blocks must be invertible.

Unlike Reed-Solomon codes, there is no general SD code construction for arbitrary \(n, m, s\) and \(r\). However, for parameters that are likely in storage systems, we do have valid constructions of SD codes. In the absence of theory, verifying that a set of parameters generates a valid SD code requires enumerating failure scenarios and verifying that each scenario can be handled. There are:

\[
\binom{n}{m} \binom{r(n-m)}{s}
\]

failure scenarios, which means that the time to verify codes blows up exponentially. For example, verifying the SD code for \(n = 24, m = 3, s = 3, r = 24\) and \(w = 32\) took roughly three days on a standard processor.

There is some theory to help us in certain cases. Blaum et al have developed verification theorems for PMDS codes, which are a subset of SD codes [5]. PMDS codes provably tolerate more failures than SD codes: \(m\) arbitrary failures per row, plus any additional \(s\) failures in the stripe. For the purposes of most storage systems, we view the additional failure protection of PMDS codes as overkill; however, when their verification theorems apply, they are faster than enumerating failure scenarios. Therefore, we use them to verify codes, and when we fail to verify that codes are PMDS, we resort to the enumeration above.

As with Reed-Solomon codes, the choice of \(w\) affects code construction — larger values of \(w\) generate more codes. However, the rule of thumb is that larger values of \(w\) also result in slower CPU performance. Therefore, we start our search with \(w = 8\), then move to \(w = 16\) and then to \(w = 32\). We focus on \(4 \leq n \leq 24\), \(1 \leq m \leq 3\), \(1 \leq s \leq 3\) and \(r \leq 24\), which encompasses the majority of situations where SD codes would apply.

We focus on two constructions. The first is when \(a_0 = 2^i\) in \(GF(2^w)\). This is called “Code \(C^{(1)}\)” by Blaum et al [5], who have developed PMDS theorems for it. We will call it “the main construction.” The second is when \(a_0\) is equal to one, but the other \(a_i\) are chosen arbitrarily. We will call these “random constructions.” Given values of \(n, m, r, s\) and \(w\), our methodology for constructing SD codes as a follows. We first test whether the main construction is SD, but not PMDS. If not, then we perform the enumeration to test whether the main construction is SD, but not PMDS. If not, we then perform a Monte Carlo search to find a random SD construction. If the Monte Carlo search fails, we leave the search for the code to be an open problem. Although we have dedicated months of CPU time to the Monte Carlo searches, many SD constructions remain open. However, we digest our results below.

We start with \(m = 1\) and \(s = 1\), and we are protecting the system from one disk and one sector failure. We anticipate that a large number of storage systems will fall into this category, as it handles the common failure scenarios of RAID-6 with much less space overhead. The main construction is PMDS so long as \(n \leq 2^w[5]\). Therefore, \(GF(2^8)\) may be used for all systems with at most 256 disks. When \(m > 1\) and \(s = 1\), the main construction is PMDS as long as \(nr \leq 2^w\). Therefore, \(GF(2^8)\) may be used while \(nr \leq 256\), and \(GF(2^{16})\) may be used otherwise.

For the remaining combinations of \(m\) and \(s\), we digest the results of our search in Figure 4. There is only PMDS theory for \(m = 1\); hence for \(m = 2\) and \(m = 3\), all of the codes are verified using enumeration. As would be expected, PMDS and SD codes for smaller values of \(n, m, r\) and \(s\) exist with \(w = 8\). As the parameters grow, we cannot find codes for \(w = 8\), and must shift to \(w = 16\). As they grow further, we shift to \(w = 32\). For all the
parameters that we have tested, the main construction is SD for $w = 32$.

The bottom line is that for parameters that are useful in today’s disk systems, there exist SD codes, often in $GF(2^8)$ or $GF(2^{16})$.

### 6 Practical Properties of SD Codes

The main property of SD codes that makes them attractive alternatives to standard erasure codes such as RAID-6 or Reed-Solomon codes is the fact that they tolerate combinations of disk and sector failures with much lower space overhead. Specifically, to tolerate $m$ disk and $s$ sector failures, a standard erasure code needs to devote $m + s$ disks to coding. An SD code devotes $m$ disks, plus $s$ sectors per stripe. This is a savings of $s(r - 1)$ sectors per stripe, or $\frac{s(r-1)}{r}$ disks per system. The savings grow with $r$ and are independent of the number of disks in the system ($n$) and the number of disks devoted to fault-tolerance ($m$). Figure 5 shows the significant savings as functions of $s$ and $r$.

Figure 5: Space savings of SD codes over standard erasure codes as a function of $s$ and $r$.

We evaluate the overhead of encoding and decoding in terms of full-stripe operations. The reason is that many storage systems do not perform erasure coding until the data has reached a threshold size, preferring replication or triplication until enough data has accumulated to mitigate the overhead of encoding [10, 22, 52]. The main overhead of encoding is the I/O, which is similar for all erasure coding schemes.

To assess the CPU overhead of encoding a stripe, we implemented encoders and decoders for both SD and Reed-Solomon codes. We use the Intel SSE2 instruction set to accelerate the performance of Galois Field arithmetic, which results in roughly a factor of ten improvement in performance over the open-source erasure coding packages tested in [40]. We ran tests on a single Intel Core i7 CPU running at 3.07 GHz. We test all values of $n$ between 4 and 24, $m$ between 1 and 3, and $s$ between 1 and 3. We also test standard Reed-Solomon coding.

For the Reed-Solomon codes, we test $GF(2^8)$, $GF(2^{16})$ and $GF(2^{32})$. For the SD codes, we set $r = 16$, and use the codes from Figure 4.

![Figure 6: CPU performance of encoding with Reed-Solomon and SD codes.](image)

The results are in Figure 6. Each data point is the average of ten runs. We plot the speed of encoding, which is measured as the amount of data encoded per second, using stripes whose sizes are roughly 32 MB. For example, when $n = 10$, $m = 2$ and $s = 2$, we employ a block size of 204 KB. That results in a stripe with 160 blocks (since $r = 16$) of which 126 hold data and 34 hold coding. That is a total of 31.875 MB, of which 25.10 MB is data. It takes 0.0225 seconds to encode the stripe, which is plotted as a rate of 25.10/0.0225 = 1116 MB/s.

The jagged lines in Figure 6 are a result of switching between $GF(2^{16})$ and $GF(2^{32})$, because the Monte Carlo search found codes in $GF(2^{16})$ for some values of $n$, but did not for others. Because the performance of $GF(2^8)$ is only marginally faster than $GF(2^{16})$, the effect of switching between $GF(2^8)$ and $GF(2^{16})$ is less noticeable.

While SD encoding is slower than Reed-Solomon coding, the speeds in Figure 6 are much faster than measurements of Plank et al. [40], which used slower Galois Field arithmetic libraries. Since those speeds were deemed fast enough to keep up with the disks of 2009, we aver that these speeds are well fast enough to keep up with current disks. Put another way, as with other current erasure coding systems (e.g. [22, 25]), the CPU is not the bottleneck; performance is limited by I/O.

To evaluate decoding, we note first that the worst case of decoding SD codes is equivalent to encoding. That is because encoding is simply a special case of decoding that requires the maximum number of equations and terms. The more common decoding cases are faster. In particular, so long as there are at most $m$ failed blocks per row of the stripe, we may decode exclusively from the $C_{j,x}$ equations. This is important, because the $C_{j,x}$ equations have only $n$ terms, and therefore require less computation and I/O than the $S_{x}$ equations. We do not
evaluate this experimentally, but note that the decoding rate for $f$ blocks using the $C_{j,x}$ equations will be equivalent to the Reed-Solomon encoding rate for $m = f$ in Figure 6.

A final property that we consider is the update penalty of the codes. This is the number of coding blocks that must be updated when a data block is modified. In a Reed-Solomon code, the update penalty achieves its minimum value of $m$. Other codes, such as EVENODD [4], RDP [12] and Cauchy Reed-Solomon [7] codes have higher update penalties. Assuming that $s \leq n - m$, the update penalty of SD codes is roughly $2m + s$, which is rather high. For that reason, these codes may be more appropriate for log-based storage systems which do not allow overwriting of data [36, 46], or cloud-based systems where stripes become immutable once they are erasure coded [10, 22].

7 Open Source Implementation

We have implemented an SD encoder and decoder in C and will post it as open source under the New BSD License. The programs allow a user to import data and/or coding information and then to perform either encoding or decoding using the techniques described above. The Galois Field arithmetic implementation leverages the SSE2 instructions for fast performance as described above in Section 6. The programs include all of the SD constructions described in Section 5 above.

Our implementation does not implement RAID or any other storage methodology. As such, we do not expect users to employ the implementation directly, but instead to use it as a blueprint for building their own SD encoded systems.

8 Related Work

The most recent work on erasure codes for storage systems has focused on improving the I/O performance of systems that tolerate multiple failures, when single failures occur [25, 50, 53], and on regenerating codes that replace lost data and coding blocks with reduced I/O for decoding [9, 13, 20, 49]. The focus of this work is on MDS codes in more classic erasure coding environments.

Non-MDS codes have been explored recently because of their reduced I/O costs and applicability to very large systems [17, 21, 28, 30]. SD codes are not strictly MDS, because they do not tolerate the failures of any $mr + s$ blocks in a stripe. However, unlike previous work, they address the heterogeneous failure modes that current disk systems exhibit.

As noted in Section 5, SD codes are a superset of PMDS codes for which some theory has been developed [5]. Since they are more stringent, there are fewer PMDS codes for the parameters that we explore, and we don’t view the additional fault-tolerance provided by PMDS codes to be overly useful in today’s storage systems. However, if more theory is developed for PMDS codes, it can be applied to SD codes as in Section 5. Blaum et al were able to discover more PMDS codes for 16-bit symbols by using a variant of $GF(2^{16})$ that is a ring rather than a field. There may be more SD codes as well in this ring than in the field. Unfortunately, the Galois Field libraries mentioned above do not support rings, so we did not employ them in our search.

A second similar code is the “diff-MDS” code from IBM [27]; however, the focus of this code is correcting bit flips in main memory instead of erasures in storage systems.

Recently, Huang et al have introduced LRC codes, which are the erasure codes used in Microsoft’s Azure cloud storage [22]. These codes are equivalent to PMDS codes for $m = 1$ and are therefore also SD codes. They were designed for a different purpose, because each block in a stripe is stored on a different disk; however, they have the same properties as SD codes. We are excited that codes with these properties are having impact in today’s large scale systems, and anticipate that further parameterizations of SD codes will have far reaching impact in cloud systems.

9 Conclusion

We have presented a class of erasure codes designed for how today’s storage systems actually fail. Rather than devote entire disks to coding, our codes devote entire disks and individual sectors in a stripe, and tolerate combinations of disk and sector failures. As such, they employ far less space for coding than traditional erasure coding solutions.

The codes are similar to Reed-Solomon codes in that they are based on invertible matrices and Galois Field arithmetic. Their constructions are composed of sets of equations that are solved using linear algebra for encoding and decoding. Their performance is not as fast as Reed-Solomon coding, but fast implementations of Galois Field arithmetic allow them to perform at speeds that are fast enough for today’s storage systems.

We have written programs that encode and decode using our codes, which we will post as open source, so that storage practitioners may employ the codes without having to understand the mathematics behind them. We are enthused that instances of these codes, developed independently by Huang et al [22], are the basis of fault-tolerance in Microsoft’s Azure cloud storage system. As such, we anticipate that these codes will have high applicability in large-scale storage installations.
References


